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CS 383
Exam 1
October 6, 2017

There are 6 numbered questions. The 6 parts of Question 1 are worth 4 points each. Questions 2 through 6 are worth 15 points each. You get one point for free.

1. Which languages are regular? You don't need to prove your answers. Write an "R" in the blank next to the description of each language you think is regular. Write "N" for any language you think is not regular. In each case the alphabet is $\Sigma=\{0,1\}$

   a. __R__Strings that end in exactly 10 1's. So 0101111111111 is in this language but 0111111111111 is not.

   b. __R__Strings with any number of 0's followed by an even number of 1's.

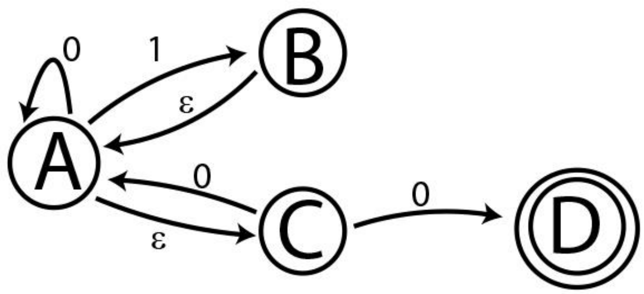
   c. __R__Strings where the digits sum to a number divisible by 5.

   d. __N__Strings where there are at least as many 0's as 1's.

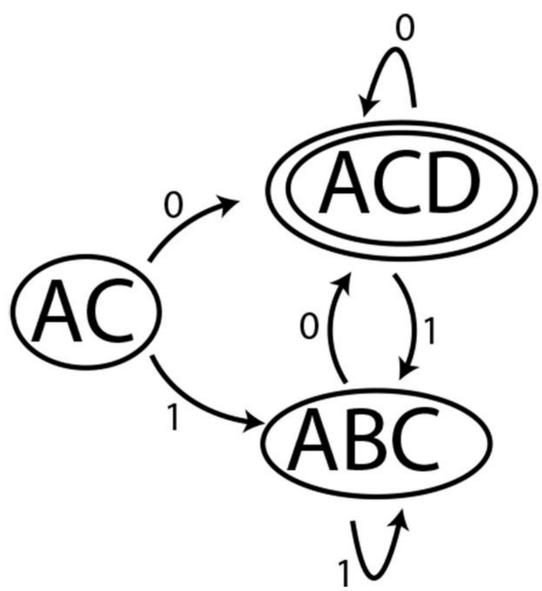
   e. __R__$0^*\mathcal{L}$ (that is the concatenation of two languages), where $\mathcal{L} = \{0^n \mid n \text{ is prime}\}$ **$0^*\mathcal{L} = \{0^n \mid n > 1\}$**

   f. __R__Strings of length 1000 that have a prime number of 1's. **This is a finite language.**

2. Here is an ε-NFA, with start state A.
   a) Convert this NFA to a DFA
   b) Describe in English the strings it accepts.



Answer:

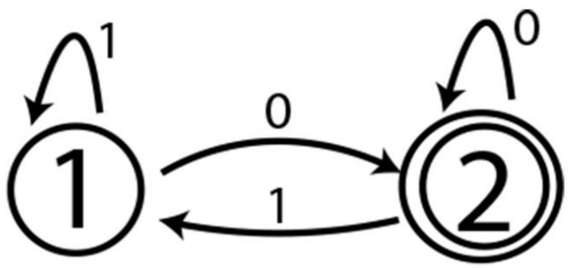


This accepts all strings ending in 0.

3. Suppose we know that for some language $\mathcal{L}$ we know that the language $00\mathcal{L} = \{00\alpha \mid \alpha \epsilon\ \mathcal{L}\}$ is regular. Must $\mathcal{L}$ be regular? Either give an example where $\mathcal{L}$ is not regular and $00\mathcal{L}$ is regular, or else show that $\mathcal{L}$ must be regular if $00\mathcal{L}$ is.

   The language $\mathcal{L}$ must be regular. Suppose $P = (\Sigma,Q,\delta,s,F)$ is a DFA accepting $00\mathcal{L}$. Let $q = \delta(s, 0)$ and let $q1=\delta(q,0)$. State q1 is where you get to in P on input 00. Let $P'= (\Sigma,Q,\delta,q1,F)$. P' is the same as P only with start state q1. Now suppose string α is in $\mathcal{L}$. Then 00α is in 00L and takes P from state s to q to q1 and then eventually to a final state. So α takes P' from q1 to a final state, and P' accepts α. Similarly, if α takes P' from q1 to a final state then 00α takes P from s to a final state, so 00α is in $00\mathcal{L}$ and α must be in $\mathcal{L}$. Altogether, the DFA P' accepts α if and only if α is in $\mathcal{L}$, so $\mathcal{L}$ is regular.

4. Consider the following DFA. We had an algorithm for converting a DFA to a regular expression. This involved making a table of regular expressions r_{ij}^{k} .



Here is the first column of a table of the r_{ij}^{k} expressions; find the 4 entries of the second column.

	k=0	k=1
r_{11}^{k}	ε+1	1*
r_{12}^{k}	0	0+(ε+1)1*0=1*0
r_{21}^{k}	1	1+11*(ε+1)=1⁺
r_{22}^{k}	ε+0	ε+0+11*0=ε+1*0

$$\boldsymbol{r_{ij}^{1} = r_{ij}^{0} + r_{i1}^{0}(r_{11}^{0})^{*}r_{1j}^{0}}$$

5. Use the pumping lemma to show carefully that the language $\{0^m1^n0^n \mid m>=2,\ n >=0\}$ is not regular.

Suppose this language is regular. Let p be its pumping constant. Let w = $0^21^p0^p$. This is a string longer than p, so it should be pumpable. Suppose w=xyz is any decomposition of w with
|xy| <= p and y nonempty. If y contains any 0s then xy^0z has fewer than 2 leading 0s and so is not in the language. If y contains any 1s then xy^2z has more 1s than trailing 0s and so is not in the language. This means there is no pumpable decomposition of w, and the language can't be regular.

6. Give a grammar for the language $\{0^n1^m \mid n > m > 0\}$

   I think of this language as $0^+\{0^m1^m \mid m>0\}$. Here is a grammar for that way of thinking of it:

   S => AB
   A => 0A | 0 (A generates 0^+)
   B => 0B1 | 01 (B generates $\{0^m1^m \mid m>0\}$)